

## **PROBLEMARIO:**

Anexar al problemario correspondiente de derivadas, los siguientes ejercicios, que se proponen:

- 25 derivadas de funciones algebraicas
- 30 derivadas de funciones trascendentes (exponenciales, logarítmicas y trigonométricas).

Cualquier duda, quedo a sus órdenes:

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9 ●●● Calcula la derivada de la función  $y = x\sqrt{x+1}$

**Solución**

Se aplica la fórmula  $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(x\sqrt{x+1}) = x \frac{d}{dx}\sqrt{x+1} + \sqrt{x+1} \frac{dx}{dx} = x \left( \frac{1}{2\sqrt{x+1}} \right) + \sqrt{x+1} = \frac{x}{2\sqrt{x+1}} + \sqrt{x+1} \\ &= \frac{x+2(x+1)}{2\sqrt{x+1}} = \frac{x+2x+2}{2\sqrt{x+1}} = \frac{3x+2}{2\sqrt{x+1}} \end{aligned}$$

Por consiguiente,  $\frac{dy}{dx} = \frac{3x+2}{2\sqrt{x+1}}$

10 ●●● Obtén la derivada de la función  $f(x) = \frac{x^2-5}{1-3x^2}$

**Solución**

Se aplica la fórmula  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$  y se obtiene:

$$f'(x) = \frac{(1-3x^2)(2x) - (x^2-5)(-6x)}{(1-3x^2)^2} = \frac{2x-6x^3+6x^3-30x}{(1-3x^2)^2} = -\frac{28x}{(1-3x^2)^2}$$

## EJERCICIO 29

Deriva las siguientes funciones:

1.  $y = -10$

2.  $y = 5$

3.  $f(x) = a^2$

4.  $s(t) = b^2$

5.  $y = 6x$

6.  $y = \frac{3}{4}x$

7.  $f(x) = ax$

8.  $s(t) = b^2t$

9.  $f(x) = 5x\sqrt{2}$

10.  $y = ax\sqrt{b}$

11.  $f(x) = x^5$

12.  $f(x) = 4x^3$

13.  $s(t) = \frac{1}{5}t^4$

14.  $y = x^{\frac{9}{2}}$

15.  $f(x) = x^{\frac{4}{3}}$

16.  $y = 6x^{\frac{3}{2}}$

17.  $f(x) = x^{\frac{2}{5}}$

18.  $f(x) = 4x^{\frac{1}{4}}$

19.  $f(x) = \sqrt{x}$

20.  $s(t) = \sqrt[4]{t}$

21.  $f(x) = 5\sqrt[3]{x}$

22.  $f(x) = \frac{x^5}{7}$

23.  $f(x) = \frac{x^4}{9}$

24.  $s(t) = \frac{t^3}{a}$

25.  $f(x) = \frac{5}{x^2}$

26.  $f(x) = \frac{2}{x^6}$

27.  $f(x) = \frac{\sqrt{x}}{2}$

28.  $s(t) = \frac{\sqrt[3]{t}}{5}$

29.  $f(x) = \frac{4}{\sqrt{x}}$

30.  $s(t) = \frac{5}{\sqrt[3]{t}}$

31.  $f(x) = \frac{4}{\sqrt[3]{x}}$

32.  $f(x) = 7x^3 - 3x^2 + 3x - 12$

33.  $f(x) = x^4 - 5x^3 + 8x^2 - x - 6$

34.  $f(x) = 5x^2 + 4x + 4mn - 2$

35.  $f(x) = 3ax^4 - 4ax^3 - 5bx^2 + 7cx$

36.  $f(x) = \frac{x^3}{6} - \frac{3x^2}{5} - \frac{4x}{9} - \frac{1}{5}$

37.  $s(t) = \frac{t^5}{6} - \frac{t^4}{5} + \frac{t^3}{4} - \frac{t^2}{7} + \frac{t}{9} - \frac{2}{3}$

38.  $f(x) = \frac{x^2}{\sqrt{a^2 + b^2}} - \frac{x}{a} + \frac{c}{b}$

39.  $s(t) = \frac{4}{t^2} - \frac{5}{t} - \frac{9}{5}$

40.  $f(x) = \frac{5}{x^4} - \frac{6}{x^3} - \frac{7}{x^2} - \frac{3}{x} + \frac{1}{5}$

41.  $s(t) = \frac{t^3}{5} - \frac{2}{t^2} + \frac{6}{t} - \frac{3}{5}$

42.  $f(x) = \frac{\sqrt[3]{x}}{5} - \frac{3}{\sqrt[3]{x}}$

43.  $f(x) = \frac{x^4 - 3x^3 - 6x^2 - 3x + 2}{x}$

44.  $f(x) = 2x^{\frac{3}{2}} + \frac{5}{2}x^{\frac{1}{2}} - \frac{4}{3}x^{-\frac{3}{2}}$

45.  $f(x) = 8\sqrt{x} + 9\sqrt[3]{x^2} + 4\sqrt[4]{x^3}$

46.  $f(x) = ax^n + bx^{n-1}$

47.  $f(x) = \frac{x^2}{3} + \frac{5x}{7} - \frac{8}{5}$

48.  $f(x) = a\sqrt[3]{x} + b\sqrt{x}$

49.  $y = \frac{1}{2}\sqrt[3]{x^2} + \frac{\sqrt[4]{x^5}}{3}$

50.  $f(x) = \frac{2}{\sqrt[4]{x^5}} - \frac{1}{\sqrt{x}} + \frac{3}{x^{-1}}$

51.  $f(x) = \frac{7}{x^{-2}} + \frac{5}{x^{-3}}$

52.  $f(x) = \frac{3}{x^2} + \frac{5}{x} - 2x$

53.  $f(x) = \frac{3x^2 + 5x + 8}{\sqrt[3]{x}}$

54.  $y = \sqrt{x^{-1}} \left( \sqrt{x} - \frac{3}{\sqrt{x}} \right)$

55.  $y = (3x - 4)^5$

56.  $y = (2 - 4x)^3$

57.  $y = (3x^6 - 2x^4)^4$

58.  $y = \left( 4x^{\frac{3}{2}} - 2x^{\frac{1}{2}} \right)^3$

59.  $y = \sqrt{5 - 3x^2}$

60.  $y = \sqrt[3]{x^3 + 2}$

61.  $y = \left( x + \frac{1}{x} \right)^{-1}$

62.  $y = \frac{2}{3}\sqrt{2x^2 + 6x}$

63.  $y = \left( \frac{x}{3} + 6\sqrt{x} \right)^3$

64.  $f(x) = \sqrt[4]{x^4 - 2}$

65.  $f(x) = (x^2 + 5x - 3)^3$

66.  $y = \sqrt[3]{(2x - 3)^2}$

67.  $y = \sqrt{\sqrt{4x + 3}}$

68.  $f(x) = \left(\frac{1}{3}x + 2\right)^3$

69.  $y = \left(\frac{2}{x} - \frac{1}{x^2}\right)^{\frac{1}{2}}$

70.  $f(z) = \sqrt{z^2 - 4}$

71.  $y = \sqrt[3]{x^6 + 3x}$

72.  $y = \left(4x^2 - \frac{1}{2}x\right)(9x + 8)$

73.  $y = (5x - 3)\left(4x - \frac{3}{x}\right)$

74.  $y = x^3(3x + 1)$

75.  $f(x) = x\sqrt{2x + 1}$

76.  $y = \frac{x}{3}(2x + 1)^3$

77.  $y = x^2\sqrt{x - 1}$

78.  $f(x) = (3x^2 - 5)^4(2x^2 + 1)^3$

79.  $f(\theta) = (\theta^2 + 1)^3(\theta^3 - 2)^2$

80.  $s = \frac{\sqrt{4 - 3t}}{t^{-1}}$

81.  $s(t) = t^3\left(\frac{2}{t} - \frac{3}{t^2}\right)^2$

82.  $f(x) = \frac{6}{2 - 4x}$

83.  $f(t) = \frac{b}{a}\sqrt{a^2 - t^2}$

84.  $f(r) = \frac{r^2 - 3}{\sqrt{r^2 - 4}}$

85.  $f(t) = \frac{6t - 3}{5t + 8}$

86.  $f(z) = \frac{6 - 3z}{5 - 6z}$

87.  $f(x) = \frac{ax + b}{ax - b}$

88.  $f(x) = \frac{2}{\sqrt{x}} - \frac{\sqrt{3x}}{3}$

89.  $f(t) = \sqrt{\frac{1 - 2t}{1 + 2t}}$

90.  $f(w) = \left(\frac{w - 3}{w + 2}\right)^2$

91.  $f(\theta) = \frac{6(2 - \theta^3)}{3 - 2\theta}$

92.  $f(s) = \frac{s^2 - 2}{s^2 - 6s}$

93.  $f(x) = \frac{5x^2}{2\sqrt{b^2 + x^2}}$

94.  $f(t) = \frac{(9t - 6)^3}{(27 - 3t)^2}$

95.  $f(x) = \frac{4xb}{2a - 6x}$

96.  $f(x) = 2x\sqrt{4 - x^2}$

97.  $y = \frac{2}{\sqrt{x^4 - a^4}}$

98.  $y = \frac{x^3}{\sqrt[3]{x^2 + 3}}$

99.  $y = (2x + 3)\sqrt{x^2 + 3x}$

100.  $y = \frac{x\sqrt{x + 1}}{x + 1}$

9.  $f(x) = 6 \sec x^2$

10.  $f(x) = \frac{1}{2} \csc \frac{x}{4}$

11.  $f(x) = a \cos 3x$

12.  $f(x) = \cot(3x - 5)$

13.  $f(x) = 2 \operatorname{sen} \frac{x}{2}$

14.  $f(x) = \cos\left(5x - \frac{\pi}{2}\right)$

15.  $s(t) = \tan(at + \pi)$

16.  $f(x) = \operatorname{sen} x + \cos x$

17.  $s(t) = \operatorname{sen} \sqrt{t}$

18.  $f(x) = \cot \sqrt[3]{x}$

19.  $f(x) = \operatorname{sen} \frac{1}{x}$

20.  $s(t) = \cos \frac{1}{t^3}$

21.  $f(x) = \sec \frac{1}{\sqrt{x}}$

22.  $f(x) = \tan 3x - 3x$

23.  $f(x) = ax + \cot ax$

24.  $f(x) = \operatorname{sen}(x - 1)^2$

25.  $s(t) = \cos(3t^2 + 2)^3$

26.  $f(x) = 4 \cot \sqrt{x-1}$

27.  $f(x) = \tan\left(\frac{x+1}{x-1}\right)$

28.  $f(x) = \sec\left(\frac{ax+b}{ax-b}\right)$

29.  $f(x) = \operatorname{sen}^2 5x$

30.  $f(x) = \cos^3 bx$

31.  $f(x) = \tan^4 3x^2$

32.  $f(x) = \sqrt{\operatorname{sen} 4x}$

33.  $f(x) = \sqrt{\sec 5x^2}$

34.  $f(x) = \sqrt[3]{\tan x^2}$

35.  $f(x) = x \operatorname{sen} x$

36.  $f(x) = x^2 \cos x^2$

37.  $f(x) = \frac{\operatorname{sen} 3x}{x}$

38.  $f(t) = \frac{\cos 5t^2}{t^2}$

39.  $y = \operatorname{sen}(ax^2)$

40.  $y = a \cos(3x)$

41.  $y = \tan \sqrt{x}$

42.  $y = \frac{1}{6} \sec 3x^2$

43.  $y = \frac{1}{2} \csc \frac{2x}{3}$

44.  $y = x^2 + 3x - \operatorname{sen}\left(\frac{1}{x}\right)$

45.  $y = -3 \cot(1 - x^2)$

46.  $y = \frac{2}{3} \operatorname{sen}\left(\frac{x+1}{x-1}\right)$

47.  $y = \operatorname{sen}^2(2bx)$

48.  $y = \tan^4(2x - 1)^3$

49.  $y = \sqrt{\sec 2x}$

50.  $y = \sqrt[3]{\tan x^2}$

51.  $y = x \cdot \cos^3 4x$

52.  $y = \frac{x^2}{\operatorname{sen} ax}$

53.  $y = x\sqrt{\csc x}$

54.  $y = \frac{\cos(mx)}{\sin(nx)}$

55.  $y = \frac{1}{1 + \sin x}$

56.  $y = x \cos x - \sin x$

57.  $y = \sqrt{\frac{\tan x - 1}{\tan x + 1}}$

58.  $y = x^2 \sin 2x - 4x \cos 2x - \sin 2x$

59.  $y = \cos(2x - 1) \cdot \tan(1 - 2x)$

60.  $y = x^2 \sec(\pi - x)$

61.  $y = \left(\frac{3x \sin x}{3x + 1}\right)^3$

62.  $y = \cos \sqrt{\frac{x+1}{x-1}}$

63.  $y = \frac{1 + \tan^2 x}{x \sec x}$

64.  $y = \frac{x(1 + \sin x)(1 - \sin x)}{\cos x}$

65.  $y = 2 \sin x \cos x$

66.  $y = \frac{\csc x \cdot \tan x}{\cos x}$

67.  $y = \sqrt{\frac{1 + \cos x}{2}}$

68.  $y = \cos^2(3x + 1) - \sin^2(3x + 1)$

69.  $y = \frac{\sqrt{1 - \sin^2 x}}{x^2}$

70.  $y = \frac{(1 + \tan x)^2}{\sec x}$

71.  $y = \frac{1}{3} \sin^3 x - \sin x + 1$

72.  $y = 2 \cos x + 2x \sin x - x^2 \cos x$

73.  $y = \frac{3}{8}x - \frac{1}{8} \sin 4x + \frac{1}{64} \sin 8x$

➔ Verifica tus resultados en la sección de soluciones correspondiente.

### Derivadas de funciones inversas trigonométricas

#### EMPLOS

Deriva la función  $y = \arcsen x^2$

#### Solución

Se aplica la fórmula  $\frac{d}{dx}(\arcsen v) = \frac{1}{\sqrt{1-v^2}} \cdot \frac{dv}{dx}$

$$\frac{dy}{dx} = \frac{d}{dx}(\arcsen x^2) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot \frac{d(x^2)}{dx} = \frac{1}{\sqrt{1-x^4}} (2x) = \frac{2x}{\sqrt{1-x^4}}$$

Por consiguiente, la derivada de la función es  $y' = \frac{2x}{\sqrt{1-x^4}}$

9.  $f(x) = \arctan \frac{x}{a}$

10.  $f(x) = 2 \arccos \sqrt{x}$

11.  $y = \arcsin(3 - x^2)$

12.  $y = \arccos \sqrt{1 - x^2}$

13.  $y = x^2 \arctan x$

14.  $y = x \arcsin x + \sqrt{1 - x^2}$

15.  $y = 8 \arccot \left( \frac{\sqrt{16 - x^2}}{x} \right) - \frac{x\sqrt{16 - x^2}}{2}$

16.  $y = x \operatorname{arccsc}(x^{-1}) + \arctan \left( \frac{x}{\sqrt{1 - x^2}} \right)$

17.  $y = \left( \frac{x^2 + 1}{2} \right) \arctan x - \frac{x}{2}$

18.  $\varphi = \operatorname{arccsc} \sqrt{\theta^2 - 1}$

19.  $y = \frac{x}{2} \sqrt{1 - 4x^2} + \frac{1}{4} \arcsin 2x$

20.  $y = \frac{x^3}{3} \arcsin x + \left( \frac{x^2 + 2}{9} \right) \sqrt{1 - x^2}$

21.  $f(r) = \sqrt{b^2 - r^2} + b \cdot \arcsin \frac{r}{b}$

22.  $y = x - \arctan x$

23.  $y = \arctan(2x) + \arcsin \left( \frac{x}{\sqrt{1 + x^2}} \right)$

24.  $y = \arcsin \sqrt{x}$

25.  $y = x \arccos \left( \frac{1}{x} \right)$

26.  $y = \arcsin(4ax - 4x^2)$

27.  $f(r) = \arcsin(r - 2)$

28.  $y = \frac{1}{4} \arctan \left( \frac{2x + 1}{2} \right)$

29.  $y = 4 \arcsin \left( \frac{x - 2}{2} \right) - \sqrt{4x - x^2}$

30.  $y = 6 \operatorname{arccsc} \left( \frac{2}{x - 2} \right) - \frac{(x + 6)\sqrt{4x - x^2}}{2}$

31.  $y = \frac{x - 1}{2} \sqrt{2x - x^2} + \frac{1}{2} \arcsin(x - 1)$

32.  $s(t) = 3\sqrt{9 - t^2} + 2 \arcsin \frac{t}{3}$

33.  $6y = 25 \arcsin \frac{3x}{5} + 3x\sqrt{25 - 9x^2}$

34.  $w = 2\sqrt{\theta + 2} + \sqrt{2} \arctan \sqrt{\frac{\theta + 2}{2}}$

35.  $y = \frac{2}{3} \arctan \left( \frac{1}{3} \tan \frac{x}{2} \right)$

36.  $y = -\frac{x}{3} + \frac{5}{6} \arctan \left( 2 \tan \frac{x}{2} \right)$

37.  $y = \arcsin \left( \cos \frac{x}{3} \right)$

38.  $y = x \operatorname{arccot}(\tan x)$

39.  $y = \frac{\operatorname{arccsc}(2x)}{\sqrt{4x^2 - 1}}$

40.  $y = \arccos \left( 2 \sec \frac{x}{2} \right)$

41.  $y = 4 \arcsin \left( \frac{2x - 4}{3x + 2} \right)$

42.  $s = t^2 \arccos(1 - t) + 2t$

43.  $y = \arccos(a + x)$

Verifica tus resultados en la sección de soluciones correspondiente . . . . .

## EJERCICIO 33

Obtén la derivada de las siguientes funciones:

1.  $y = \ln x^3$

2.  $f(x) = \ln 4x^2$

3.  $f(x) = \ln(3x^2 - 5x + 2)$

4.  $f(x) = \ln \sqrt{x}$

5.  $f(x) = \log x^6$

6.  $f(x) = \log 5x^3$

7.  $f(x) = \log_3 x$

8.  $f(x) = \log_4 \sqrt[3]{x}$

9.  $f(x) = \ln^4 x$

10.  $f(x) = \ln^3 5x$

11.  $y = x^2 \ln x$

12.  $y = x \ln x^2$

13.  $y = \frac{\ln x}{x}$

14.  $f(x) = \frac{\ln x^2}{x}$

15.  $y = \ln \sqrt{b - ax}$

16.  $f(x) = \ln x^2 \sqrt{3x^2 - 1}$

17.  $f(x) = \ln ax \sqrt{ax^2 - b}$

18.  $y = \ln \left( \frac{3x-5}{2x+1} \right)$

19.  $y = \ln \sqrt{\frac{cx-b}{cx+b}}$

20.  $y = \ln \operatorname{sen} x$

21.  $y = \ln \cos 5x$

22.  $y = \ln(x^2 - 4)$

23.  $y = \ln \sqrt{3x+4}$

24.  $y = \ln \left( \frac{2x-3}{2x+3} \right)$

25.  $y = \ln \sqrt[3]{x^3 + 8}$

26.  $y = \ln^2(\sqrt{x})$

27.  $y = \ln[(6x+4)(3x^2+2)]$

28.  $y = \log_3 \sqrt{\frac{1-2x}{1+2x}}$

29.  $y = \log(5bx^3 - 3\sqrt{x})$

30.  $y = x - \ln(e^x \cos x)$

31.  $y = \ln(\operatorname{sen}^2 x)$

32.  $y = x \ln x$

33.  $y = \ln(\sec^2 2x \cdot \cos^3 2x)$

34.  $y = \sqrt{\ln x}$

35.  $y = \ln(\sec x + \tan x)$

36.  $y = \ln \sqrt{1 - \operatorname{sen} 2x}$

37.  $y = \ln(x \operatorname{sen} x)$

38.  $y = x^3 \ln x^2$

39.  $y = \ln(\tan \sqrt{x})^3$

40.  $y = \log \sqrt{x}$

41.  $y = 2^{x^2+5x}$

42.  $f(x) = b^{\sqrt{x}}$

43.  $y = 3^{\ln x}$

44.  $y = 5^{x \operatorname{sen} x}$

45.  $y = x \cdot 2^{\ln x}$

46.  $y = x \cdot 5^x$

47.  $y = e^{x^2}$

48.  $y = e^{3x^2 - 2x + 1}$



## 4 CAPÍTULO

MATEMÁTICAS SIMPLIFICADAS

$$49. y = e^{\sqrt{3x^2-1}}$$

$$50. y = e^{x \tan x}$$

$$51. y = \frac{b}{2} \left( e^{\frac{2x}{b}} - e^{-\frac{2x}{b}} \right)$$

$$52. y = \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$$

$$53. f(x) = e^{4x}$$

$$54. f(x) = e^{5x^2}$$

$$55. f(x) = e^{3x-1}$$

$$56. f(x) = e^{\frac{x}{5}}$$

$$57. f(t) = \sqrt[3]{e^t}$$

$$58. f(x) = \sqrt[4]{e^x}$$

$$59. f(x) = e^{\frac{1}{x^2}}$$

$$60. f(x) = e^{\sqrt{x}}$$

$$61. f(\theta) = e^{\sin^2 \theta}$$

$$62. f(x) = e^{\cos 2x}$$

$$63. y = e^{x \sin x}$$

$$64. f(x) = 5^{3x}$$

$$65. f(x) = 7^{2x}$$

$$66. f(x) = 5^{x^2}$$

$$67. y = x^{2x}$$

$$68. y = x^{\cos x}$$

$$69. y = \sqrt{x}$$

$$70. y = e^{\arctan x}$$

$$71. y = \ln(\sqrt{xe^{2x}})$$

$$72. y = xe^{\ln x^2}$$

$$73. y = \frac{e^x}{x+1}$$

$$74. y = \frac{xe^x}{\ln x^2}$$

$$75. y = \sqrt{\frac{\ln x+1}{\ln x-1}}$$

$$76. y = \sqrt{\frac{e^{\sin x}+1}{e^{\sin x}-1}}$$

$$77. y = \ln(\ln \sin^2 ax)$$

$$78. y = e^{\ln \sqrt{e^{\sin x}}}$$

$$79. y = x^2 e^{\sin x}$$

$$80. y = \frac{\ln \sin^x x}{x}$$

$$81. y = \ln(3ax^2 \sqrt{x^2-4})$$

$$82. y = \sqrt{x^2+9} + 3 \ln(x + \sqrt{x^2+9})$$

$$83. y = \frac{1}{4} \sec 2x \tan 2x + \frac{1}{4} \ln(\sec 2x + \tan 2x)$$

$$84. y = x \arctan x - \ln \sqrt{1+x^2}$$

$$85. y = \frac{x}{2} \sqrt{x^2-4} - 2 \ln(x + \sqrt{x^2-4})$$

$$86. y = x \operatorname{arcsec} x - \ln(x + \sqrt{x^2-1})$$

$$87. y = \frac{1}{12} \ln \left( \frac{2x-3}{2x+3} \right)$$

$$88. y = x \operatorname{arccot} x + \ln \sqrt{1+x^2}$$

$$89. y = x \operatorname{arccsc} \frac{x}{2} + 2 \ln(x + \sqrt{x^2-4})$$

Verifica tus resultados en la sección de soluciones correspondiente.